

PERIODIC TEST-1 2024

APPLIED MATHEMATICS (241)

Class: XII Commerce

Duration: 1 Hour

Date: 03/08/24

MARKING SCHEME

Max. Marks: 25

Q No.	Answer	Scheme
1	C	$2x^{2x}(1+\log x)$
2	A	$\frac{1}{t}$
3	B	$\frac{1}{\pi}$
4	A	$\left(\frac{1}{3}, \infty\right)$
5	A	
6		$Y = (x)^{\log x}$, $\log y = \log x \log x = (\log x)^2$ $\frac{1}{y} \frac{dy}{dx} = 2 \log x \cdot \frac{1}{x}$ $\frac{dy}{dx} = y(2 \log x)/x$ $\frac{dy}{dx} = (x)^{\log x} 2 \log x / x$
7		$\log x + \log y = x^2 + y^2$ $1/x + 1/y \frac{dy}{dx} = 2x + 2y \frac{dy}{dx}$ $2 \frac{dy}{dx} - \frac{1}{y} \frac{dy}{dx} = \frac{1}{x} - 2x$ $\frac{dy}{dx} \left(2 - \frac{1}{y}\right) = \frac{1}{x} - 2x$ $\frac{dy}{dx} = \frac{y(2x^2 - 1)}{x(1 - 2y^2)}$
8		<p>Let r be the radius and A cm² be the disturbed area at time t second.</p> $A = \pi r^2$ $\frac{dr}{dt} = 3.5 \text{ cm/sec}, \frac{dA}{dt} = \frac{d}{dt} \pi r^2 = 2\pi r \frac{dr}{dt} = 2\pi \times 7.5 \times 3.5 = 165 \text{ cm}^2$
9		<p>Average cost = $C(x)/x = 0.005x^2 - 0.02x + 30 + \frac{5000}{x}$</p> <p>Marginal cost = $\frac{dC}{dx} = 0.015x^2 - 0.04x + 30$.</p>
10		$Y = (2x+3)^{x-5}, \log y = (x-5) \log (2x+3)$ $\frac{1}{y} \frac{dy}{dx} = (x-5) \frac{2}{2x+3} + \log (2x+3) \cdot 1$ $\frac{dy}{dx} = (2x+3)^{x-5} \left[\frac{2(x-5)}{2x+3} + \log(2x+3) \right]$
11		$e^y \frac{d}{dx} (x+1) + (x+1) \frac{d}{dx} e^y = 0$ $e^y + (x+1) e^y \frac{dy}{dx} = 0$

	$\frac{dy}{dx} = -e^y$
12	$AC = C/x = \frac{x}{90} + 2 + \frac{1000}{x}$ $\frac{d}{dx} A = \frac{1}{90} - \frac{1000}{x^2} = 0$ $x = 3000$ <p>Now, $\frac{d^2 A}{dx^2} = \frac{2000}{x^3} > 0$ when $x=300$</p> <p>Therefore average cost is minimum when $x=300$</p> <p>Minimum average cost = $26/3$</p>
13	$P(x) = R(x) - C(x)$ $= \frac{597}{2}x - \frac{3}{4}x^2 - 50$ $\frac{dP}{dx} = \frac{597}{2} - \frac{6}{5}x = 0$ $x = 199$ $\frac{d^2 P}{dx^2} = -\frac{6}{4} < 0 \text{ when } x = 199$ <p>Therefore maximum profit at $x = 199$</p> <p>Maximum profit = 29650.75</p>